

PROBLEM SET #1

MA 109C: INTRODUCTION TO GEOMETRY AND TOPOLOGY

Due on Tuesday April 16, 2024

(Last modified April 10, 2024)

Solve the following problems from the textbook *Differential Topology* by V. Guillemin and A. Pollack. You may only use the relevant course material described below.

- Chapter 1, section 1, exercises 16, 17
- Chapter 1, section 2, exercises 9, 10, 11, 12
- Chapter 1, section 3, exercises 2, 9, 10
- Chapter 1, section 4, exercises 7, 11, 13
- Chapter 1, section 5, exercises 4, 5, 7, 10

Solve the following additional problem.

- Determine whether the following statement is true. Let X and Z be submanifolds of the manifold Y . Denote $\dim X = k$, $\dim Z = l$, $\dim Y = n$, and assume $k + l \geq n$. If $X \bar{\cap} Z$, then for each $p \in X \cap Z$, there exists a local coordinate system $(x_1, \dots, x_n)$ of a neighborhood U of p in Y so that

$$U \cap X = \{q \in U : x_{k+1}(q) = \dots = x_n(q) = 0\},$$

$$U \cap Z = \{q \in U : x_1(q) = \dots = x_{n-l}(q) = 0\}.$$

In words, the transversal intersection $X \bar{\cap} Z$ is locally equivalent to the “canonical transversal intersection”

$$(\mathbb{R}^k \times \{0\}^{n-k}) \bar{\cap} (\{0\}^{n-l} \times \mathbb{R}^l) \text{ in } \mathbb{R}^n.$$

Relevant course material for this homework: Up to Section 5 of Chapter 1.

You may not search the solutions to the homework problems from any source and you must write the homework in your own words. If you collaborated with other students, you must list their names on the solutions.