

PROBLEM SET #4

MA 109A: INTRODUCTION TO GEOMETRY AND TOPOLOGY

Due on Friday November 7, 2025

(Last modified October 25, 2025)

Since it's midterm examination week, let's keep the problem set short and simple.

(20 points in total)

- (1) (5 points) Let $p : X \rightarrow Y$ be a closed continuous surjective map. Show that if X is normal, so is Y .¹
- (2) (5 points) Show that every locally compact Hausdorff space is regular.
- (3) (5 points) Let X be a compact Hausdorff space. Show that X is metrizable if and only if X has a countable basis.
- (4) (5 points) Are compact manifolds metrizable? If yes, provide a proof.² If not, provide a counterexample.

Relevant course material for this homework: Up to section 36 of Munkres.

You may not search the solutions to the homework problems from any source. Please do not consult Large Language Models and please write the homework in your own words, as that is the best way to learn. If you collaborated with other students, please list their names on the solutions.

¹Hint: if U is an open set in X containing $p^{-1}(y)$, then there is a neighborhood W of y such that $p^{-1}(W) \subset U$.

²If the manifold is smooth, you can actually give it a *Riemannian* metric, whether it is compact or noncompact. Come to Ma109c to learn more about what you can do under the smoothness assumption!