

PROBLEM SET #1

MA 109C: INTRODUCTION TO GEOMETRY AND TOPOLOGY

Due on Friday April 11, 2025

(Last modified March 25, 2025)

Solve the following problems from the textbook *Differential Topology* by V. Guillemin and A. Pollack. You may only use the relevant course material described below.

(25 points in total)

- (5 points) Chapter 1, section 1, exercise 16
- (5 points) Chapter 1, section 2, exercise 10
- (5 points) Chapter 1, section 3, exercise 2
- (5 points) Chapter 1, section 4, exercise 7
- (5 points) Chapter 1, section 5, exercise 10
- (Bonus problem, 5 points) Determine whether the following statement is true. Let X and Z be submanifolds of the manifold Y . Denote $\dim X = k$, $\dim Z = l$, $\dim Y = n$, and assume $k + l \geq n$. If $X \pitchfork Z$, then for each $p \in X \cap Z$, there exists a local coordinate system $(x_1, \dots, x_n)$ of a neighborhood U of p in Y so that

$$U \cap X = \{q \in U : x_{k+1}(q) = \dots = x_n(q) = 0\},$$

$$U \cap Z = \{q \in U : x_1(q) = \dots = x_{n-l}(q) = 0\}.$$

In words, the transversal intersection $X \pitchfork Z$ is locally equivalent to the “canonical transversal intersection”

$$(\mathbb{R}^k \times \{0\}^{n-k}) \pitchfork (\{0\}^{n-l} \times \mathbb{R}^l) \text{ in } \mathbb{R}^n.$$

Relevant course material for this homework: Up to Section 5 of Chapter 1.

You may not search the solutions to the homework problems from any source and you must write the homework in your own words. If you collaborated with other students, you must list their names on the solutions.

I strongly recommend that you solve, but not submit solutions for, the following additional problems.

- Chapter 1, section 1, exercise 17
- Chapter 1, section 2, exercises 9, 11, 12
- Chapter 1, section 3, exercises 9, 10
- Chapter 1, section 4, exercises 11, 13
- Chapter 1, section 5, exercises 4, 5, 7