

PROBLEM SET #3

MA 109C: INTRODUCTION TO GEOMETRY AND TOPOLOGY

Due on Friday April 25, 2025

Solve the following problems from the textbook *Differential Topology* by V. Guillemin and A. Pollack. You may only use the relevant course material described below.

- (1) Chapter 1, section 8, exercise 14
- (2) Chapter 2, section 1, exercises 7
- (3) Chapter 2, section 1, exercises 8
- (4) Chapter 2, section 3, exercises 12
- (5) Chapter 2, section 3, exercises 16
- (6) Chapter 2, section 3, exercises 18

Relevant course material for this homework: Up to section 3 of Chapter 2.

You may not search the solutions to the homework problems from any source and you must write the homework in your own words. If you collaborated with other students, you must list their names on the solutions.

I strongly recommend that you solve, but not submit solutions for, the following additional problems.

- Chapter 2, section 1, exercises 1, 2
- Chapter 2, section 2, exercise 7
- Chapter 2, section 3, exercises 5, 7, 17, 19
- (Additional problem) In Section 1 of Chapter 1, a function $f : X(\subseteq \mathbb{R}^N) \rightarrow \mathbb{R}^M$ is said to be smooth if it is locally extendable to a smooth function, i.e., every $x \in X$ has an open neighborhood U_x in $\mathbb{R}^N$ on which a smooth (in the usual sense) function $F_x : U_x \rightarrow \mathbb{R}^M$ is defined so that $F_x|_{X \cap U_x} = f|_{X \cap U_x}$. However, in class, we used a seemingly stronger definition, namely that there is an open neighborhood U of X in $\mathbb{R}^N$ on which a smooth function $F : U \rightarrow \mathbb{R}^M$ is defined so that $F|_X = f$. Show that these two definitions are equivalent. (Hint: when constructing partitions of unity in section 8 of Chapter 1, note that the partition of unity is locally finite in an open neighborhood W of X , i.e., the functions $\theta_i : X \rightarrow [0, 1]$ are restrictions of functions $\tilde{\theta}_i : W \rightarrow [0, 1]$ to X , and the $\tilde{\theta}_i$'s are locally finite in W).